Inertial particle pair diffusion has received much less attention than fluid particle pair diffusion [1]; only the DNS work of Bec et al [2] has been reported in recent times. This is especially surprising because inertial particle transport is arguably more relevant to real world applications, such as dust clouds, sand storms, soot, and pollens dispersion. A new non-local theory of fluid particle pair diffusion has recently been proposed [3]; but the question is, can non-locality be extended to inertial particle pair diffusion? Here we derive some scaling laws for inertial pair diffusion in the limit of Stokes’ drag where inertial particle transport is given by,

\[
\frac{dx}{dt} = v(t), \quad \frac{dv}{dt} = -\frac{1}{\tau} (v(t) - u)
\]

and then we investigate it numerically using Kinematic Simulations (KS) [4,5]. In equation (1), \(x(t)\) is the particle position at time \(t\), \(v(x,t)\) is the Eulerian velocity field generated by the KS model, \(\tau\) is the particle response time. The Stokes number is, \(S_{t} = \tau / t_{h}\), where \(t_{h}\) is the Kolmogorov time scale. The root mean square separation at time \(t\) is, \(\sigma(t) = \langle l(t)^{2} \rangle^{\frac{1}{2}}\), where \(l(t) = |x_{1}(t) - x_{2}(t)|\) is the distance between particles in a pair. It is assumed that an ensemble of particle pairs is released at time \(t=0\) such that \(|l|=0 < \eta; \eta\) is the Kolmogorov scale of the turbulence. Of principle interest is the inertial particle pair diffusivity \(K_{l}\).

For short times, the energy in the small scales of turbulence is too small to affect the inertial particle relative motion, thus we expect ballistic motion at short times, \(K_{l} \sim \sigma_{l}^{3}\), for \(t < T_{\epsilon}\), where \(T_{\epsilon}\) is some time scale (to be determined). As the pair separation increases, the turbulent energies of larger size and energy will eventually become dominant and we expect the inertia pair diffusion to assimilate towards fluid particle pair diffusion, i.e. \(K_{l} \rightarrow K \sim \sigma_{l}^{1.53}\), for \(T_{\epsilon} < t < T_{\kappa}\) [3] where \(T_{\kappa}\) is a timescale when the pair diffusion separation is so large that their motions are independent. We also expect an intermediate transition regime which is determined by when the local Stokes number, \(S_{t}(t)=\tau / T_{\kappa}\). Assuming non-locality this gives the scaling, \(\sigma_{l}/\eta \sim S_{t}^{2/3}\), at the scale \(\sigma_{l}\) at which turbulent energy starts to dominate over the particle inertia [2]. However, non-locality should yield, \(\sigma_{l} / \eta \sim S_{t}^{\alpha}\), with \(\alpha>2/3\). KS was used in a frame of reference moving with the (virtual) large scale sweeping velocities, by imposing the Kolmogorov spectrum, \(E(k) \sim k^{-5/3}\), for \(1 \leq k \leq 10^{4}\), and \(E(k)=0\), for \(k < 1\). Fig. 1 shows the inertial pair diffusivity \(K_{l}/\eta \nu_{t}\), against the pair separation \(\sigma_{l}/\eta\) for a wide range of Stokes number, \(0 \leq S_{t} \leq 50\). The results in Fig. 1 display short time ballistic regimes, and long-time non-local scaling, \(K_{l} \sim \sigma_{l}^{1.53}\), and intermediate transition regimes. Fig. 2 shows \(\sigma_{l} / \eta\) against \(S_{t}\) displaying a non-local scaling slightly greater than \(2/3\). This work will be completed for different spectra \(E(k) \sim k^{-\rho}\), for \(1 < \rho \leq 3\), and for different sizes of the inertial subrange, up to \(1 \leq k \leq 10^{4}\).

References