TURBULENT INERTIAL PARTICLE PAIR DIFFUSION

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Inertial particle pair diffusion has received much less attention than fluid particle pair diffusion [1]; only the DNS work of Bec et al [2] has been reported in recent times. This is especially surprising because inertial particle transport is arguably more relevant to real world applications, such as dust clouds, sand storms, soot, and pollens dispersion. A new non-local theory of fluid particle pair diffusion has recently been proposed [3]; but the question is, can non-locality be extended to inertial particle pair diffusion? Here we derive some scaling laws for inertial pair diffusion in the limit of Stokes' drag where inertial particle transport is given by,

$$\frac{d\boldsymbol{x}}{dt} = \boldsymbol{v}(t), \qquad \frac{d\boldsymbol{v}}{dt} = -\frac{1}{\tau}(\boldsymbol{v}(t) - \boldsymbol{u}) \tag{1}$$

and then we investigate it numerically using Kinematic Simulations (KS) [4,5]. In equation (1), $\boldsymbol{x}(t)$ is the particle position at time t, $\boldsymbol{v}(t)$ is the particle velocity, $\boldsymbol{u}(\boldsymbol{x},t)$ is the Eulerian velocity field generated by the KS model, τ is the particle response time. The Stokes number is, $St= \tau/t_h$, where t_h is the Kolmogorov time scale. The root mean square separation at time t is, $\sigma_l(t)=<l(t)^2>^{½}$, where $l(t)=|\boldsymbol{x}_1(t)-\boldsymbol{x}_2(t)|$ is the distance between particles in a pair. It is assumed that an ensemble of particle pairs is released at time $t=0$ such that $l(t=0) =l_0 < \eta$. η is the Kolmogorov scale of the turbulence. Of principle interest is the inertial particle pair diffusivity $K_I(l)$.

For short times, the energy in the small scales of turbulence is too small to affect the inertial particle relative motion, thus we expect ballistic motion at short times, $K_I \sim \sigma_l^{\ 1}$, for $t << T_s$, where T_s is some time scale (to be determined). As the pair separation increases, the turbulent energies of larger size and energy will eventually become dominant and we expect the inertia pair diffusion to asymptote towards *fluid* particle pair diffusion, i.e. $K_I \rightarrow K \sim \sigma_l^{\ 1.53}$, for $T_s <<t<< T_L$, [3] where T_L is a timescale when the pair diffusion separation is so large that their motions are independent. We also expect an intermediate transition regime which is determined by when the local Stokes number, $St(l^*)=\tau/T_s=1$. Assuming non-locality this gives the scaling, $\sigma_l^{\ *}/\eta \sim St^{2/3}$, at the scale σ_l^* at which turbulent energy starts to dominate over the particle inertia [2]. However, non-locality should yield, $\sigma_l^{\ *}/\eta \sim St^{\ \alpha}$, with $\alpha > 2/3$. KS was used in a frame of reference moving with the (virtual) large scale sweeping velocities, by imposing the Kolmogorov spectrum, $E(k) \sim k^{-5/3}$, for $1 \leq k \leq 10^4$, and $E(k)=0$, for $k <1$. Fig. 1 shows the inertial pair diffusivity $K_I/\eta v_\eta$ against the pair separation σ_l/η for a wide range of Stokes number, $0 \leq St \leq 50$. The results in Fig. 1 display short time ballistic regimes, and long-time non-local scaling, $K_p \rightarrow K \sim \sigma_l^{\ 1.53}$, and intermediate transition regimes. Fig. 2 shows σ_l^*/η against St, displaying a non-local scaling slightly greater than $2/3$. This work will be completed for different spectra $E(k) \sim k^{-p}$, for $1 <p \leq 3$, and for different sizes of the inertial subrange, up to $1 \leq k \leq 10^4$.

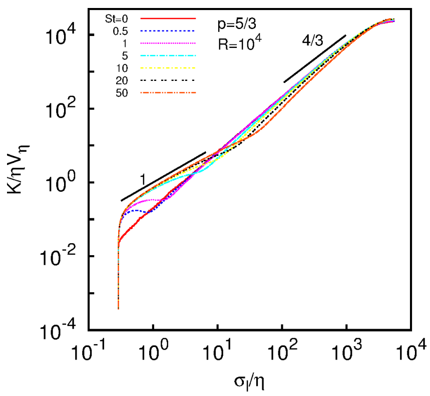

Figure 1. Log-log of $K_l/\eta v_\eta$ against σ_l/η.

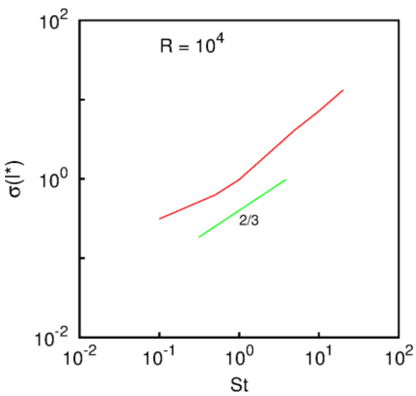

Figure 2. Log-log of σ_l^*/η against St.

References

[1] Richardson L. F. Atmospheric diffusion shown on a distance-neighbor graph. *Proc. Roy. Soc. Lond. A* **100**, 709-737 (1926).
[2] Bec, J., Biferale, L., Lanotte, A., Scagliarini, A. and F., Toschi, Turbulent pair dispersion of inertial particles, J. Fluid Mech. 645, 497. (2010).
[3] Malik N. A. Ninety years in the hunt for turbulent pair diffusion laws. IMA Conf. Turbulence, Waves and Mixing. 6-8 July, Cambridge (2016).
[4] Fung J. C. H., Hunt J. C. R., Malik N. A. & Perkins R. J. *J. Fluid Mech.* **236**, 281 (1992).
[5] Malik, N. A. Residual sweeping effect in turbulent particle pair diffusion in a Lagrangian diffusion model. *Submitted (2016)*.