A NON-LOCAL THEORY OF TURBULENT PARTICLE PAIR DIFFUSION

Nadeem A. Malik

King Fahd University of Petroleum and Minerals,
Department of Mathematics, P.O. Box 5046, Dhahran 31261, Saudi Arabia.
Email: namalik@kfupm.edu.sa ; nadeem_malik@cantab.net

Richardson's theory of turbulent particle pair diffusion [1] is based upon observational data and the hypothesis of locality, which leads to the turbulent pair diffusivity K scaling, $K \sim \sigma_l^{4/3}$, where $\sigma_l(t)$ is the root mean square pair diffusion at time t, $\sigma_l(t)=<l(t)^2>^{\frac{1}{2}}$, and $l(t)=|\mathbf{x}_1(t)-\mathbf{x}_2(t)|$ is the distance between particle pair locations. It is assumed that an ensemble of particle pairs is released at time $t=0$ such that $l(0)=l_0<\eta$. But Richardson's locality scaling has never been proven, as observed by Salazar & Collins [2], *" ... there has not been an experiment that has unequivocally confirmed R-O scaling over a broad-enough range of time and with sufficient accuracy".* Furthermore, a reappraisal of the 1926 dataset reveals that one of the data-points is from a molecular diffusion context, Fig.1 (red dot); the remaining data from geophysical turbulence display an unequivocal non-local scaling, $K \sim \sigma_l^{1.564}$, Fig.1 (black dots), [3]. A locality scaling of 4/3 (blue line) is not the best fit to the data.

Interestingly, KS [4] yields scalings which deviate from locality – in the past this has been attributed to errors in KS [5,6,7]. But in view of the above results, we have re-examined and quantifying the sweeping errors in KS. It is shown through a novel analysis based upon analyzing pairs of neighboring particle trajectories in a frame of reference moving with the large energy containing scales of motion that the errors caused by the lack of sweeping in KS is negligible even for very large inertial subranges [8]. From KS simulations containing extended inertial subrange the relative sweeping error, e_K, in the pair diffusivity K, relative to K itself, decreases with increasing pair separation, σ_l, such that $e_K \rightarrow 0$ as $\sigma_l/\eta \rightarrow \infty$, where η is the Kolmogorov turbulence microscale, and $\sigma_l(t)$ is the root mean square pair separation at time t, $\sigma_l(t)=<l(t)^2>^{\frac{1}{2}}$, where $l(t)=|\mathbf{x}_1(t)-\mathbf{x}_2(t)|$ is the distance between particle pair locations. It is assumed that an ensemble of particle pairs is released at time $t=0$ such that $l(0)=l_0<\eta$.

In a frame of reference moving with the large scales, and with Kolmogorov energy spectrum, $E(k) \sim k^{-5/3}$, for $1 \le k \le 10^6$, and $E(k)=0$, for $k<1$, the pair diffusion scaling observed in KS is, $K \sim \sigma_l^{1.53}$, Fig. 2 (red line), which differs from Richardson-Obukhov locality scaling $K \sim \sigma_l^{4/3}$. Even when a large scale eddies is included in KS, $E(k)=E_0$ at $k=k_0<1$, the error remains small in most of the inertial range, Fig. 2 (green line). (No scale is shown on the vertical axis because the energies in the two cases have been shifted vertically for direct comparison. This does not affect the slopes.)

But if the KS pair diffusion scaling is not caused by the lack of physical sweeping, as previously thought, then what is it due to? One possibility is that the KS scaling is correct -- which implies that the founding assumption of locality itself may be in error. Such a possibility has never been considered before; but it is important to note that Richardson's scaling has never been proven, as observed by Salazar & Collins [9], *" ... there has not been an experiment that has unequivocally confirmed R-O scaling over a broad-enough range of time and with sufficient accuracy".* Our results suggest that a non-local theory of turbulent pair diffusion is a viable alternative to Richardson's locality hypothesis.

From this idea, a new non-local theory based upon the principle that both local and non-local diffusional processes govern pair diffusion in homogeneous turbulence has been developed. Using a novel mathematical approach based upon the Fourier decomposition of the pair *relative velocity*, the theory is developed in the context of generalized power law energy spectra over extended inertial subranges, $E(k) \sim k^{-p}$, for $1 \le k \le R_k$, and for $1<p \le 3$. The theory predicts the scaling, $K \sim \sigma_l^{\gamma_p}$, with γ_p intermediate between the purely local and the purely non-local scalings, i.e. $(1+p)/2<\gamma_p \le 2$. Kinematic Simulations [4,5] is used to examine the predictions of the new non-local theory. For the pair diffusivity KS produces the scalings, $K \sim \sigma_l^{1.545}$ to $\sigma_l^{1.57}$, in the accepted range of *intermittent* turbulence spectra, $E(k) \sim k^{-1.72}$ to $k^{-1.74}$; Fig. 3 shows the case for $k^{-1.74}$ which is in remarkably close agreement with the revised 1926 dataset, Fig. 1. Note that $K \sim \sigma_l^{1.57}$ is equivalent to $<l(t)^2> \sim t^{4.65}$.

The consequences of non-locality for the general theory of turbulence is the subject of active investigation by the author.

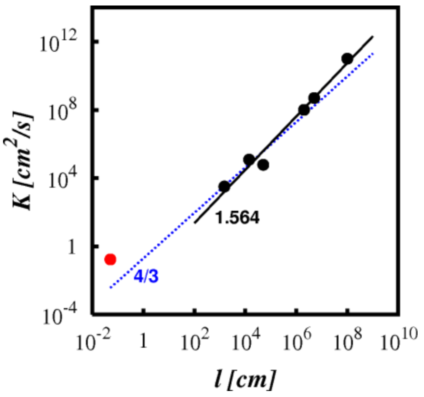

Figure 1. Log-log of pair diffusivity against separation: Richardson (Red). Current reappraisal (Black) [3].

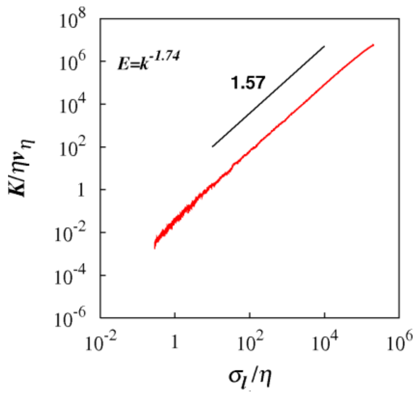

Figure 3. Log-log of pair diffusivity against separation, for KS energy spectrum $E(k) \sim k^{-1.74}$, [3].

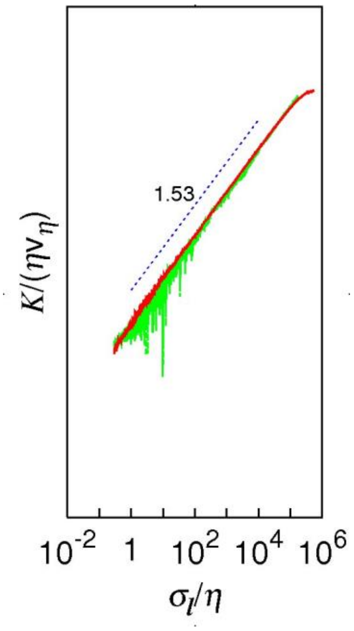

Figure 2. Log-log plot of the turbulent diffusivity, $K/\eta v_\eta$, against, σ_l/η, from KS with spectrum, $E(k) \sim k^{-5/3}$, for $1 \leq k \leq 10^6$ (red line); and the (green line) is with additional energy in the large scales, $E(k)=E_0$ at $k = k_0 < 1$. A slope of 1.53 is shown for comparison.

Acknowledgement: The author gratefully acknowledges funding from SABIC through grant # SB101011.

References

[1] Richardson L. F. Atmospheric diffusion shown on a distance-neighbour graph. *Proc. Roy. Soc. Lond. A* **100**, 709-737 (1926).

[2] Salazar & Colins, B. J. Two-particle dispersion in isotropic turbulent flows. J. Fluid Mech. **526**, 277 (2005).

[3] Malik N. A. On turbulent particle pair diffusion. *Submitted (2016). http://arxiv.org/abs/1405.3625v3*.

[4] Fung J. C. H., Hunt J. C. R., Malik N. A. & Perkins R. J. Kinematic simulation of homogeneous turbulence by unsteady random Fourier modes. *J. Fluid Mech.* **236**, 281 (1992).

[5] Thomson, D. J. & Devenish, B. J. Particle pair separation in kinematic simulations. *J. Fluid Mech.* **26**, 277--302 (2005).

[6] Nicolleau, F. C. G. A. & Nowakowski, A. F. Presence of a Richardson's regime in kinematic simulations. *Phys. Rev.* ***E*** 83, 056317 (2011).

[7] Eyink, G. L. & Benveniste, D. Suppression of particle dispersion by sweeping effects in synthetic turbulence. *Phys. Rev. E* **87**, 023011 (2013).

[8] Malik, N. A. Residual sweeping effect in turbulent particle pair diffusion in a Lagrangian diffusion model. *Submitted (2016)***.** *http://arxiv.org/abs/1501.07186*.

[9] Salazar & Colins, B. J. Two-particle dispersion in isotropic turbulent flows. J. Fluid Mech. **526**, 277 (2005).