

Random walks with long range steps generated by fractional Laplacian matrices: recurrence behavior and emergence of Levy flights

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We discuss a Markovian random walk on undirected regular networks generated by power matrix functions ('Fractional Laplacian matrices') in the admissible index range $0 < \alpha \leq 2$ and refer this walk to as to as 'Fractional Random Walk' (FRW) [1]. By analyzing convergence behavior of the probability generating functions (Lattice Green's functions) for the FRW, we establish a generalization of Polya's recurrence theorem for Fractional Random Walks on d -dimensional infinite lattices: The Fractional Random Walk is transient for dimensions $d > \alpha$ and recurrent for $d \leq \alpha$ the lattice. As a consequence the FRW is transient for $0 < \alpha < 1$ in all lattice dimensions $d = 1, 2, \dots$ and in the range $1 < \alpha \leq 2$ for dimensions $d > 2$. Finally, for $\alpha = 2$ Polya's classical recurrence theorem is recovered, namely the walk is transient for lattice dimensions $d > 2$ and otherwise recurrent [3]. The generalization of Polya's recurrence theorem remains valid for the class of random walks with Levy flight asymptotics for long-range steps. We also analyze mean first passage probabilities, mean first passage times, global mean first passage times (Kemeny constant) and mean occupation times for nodes. For the infinite 1D lattice (infinite ring) we obtain for the transient regime $0 < \alpha < 1$ closed form expressions for the lattice Green's function containing the information the of mean occupation times of the walker spends on a node. Generally the entries of the Green's function of the FRW (the mean occupation times of nodes for infinite times of observation) fulfill in the transient regime $d > \alpha$ Riesz potential decay for nodes far away from the departure node. The non-locality of the FRW generates a remarkably rich dynamics interesting for a wide range of interdisciplinary applications such as random walk based search strategies. The main dynamic effects of the FRW is generated by the non-diagonality of the fractional Laplacian matrix with asymptotic Levy type heavy tailed inverse power law decay for the probability of long-range moves [1,4,5].

References

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